Unless explicitly requested by a problem, do not include sketches as part of your proof. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

**Problem 1** (2 pts)**.** Fix an integer $n \geqslant 2$. Prove that a sequence of integers $d_1, \dots, d_n$ is the degree sequence of some tree if and only if $d_i \geqslant 1$ for all $i \in \{1, \dots, n\}$ and $\sum_{i=1}^{n} d_i = 2n - 2$.

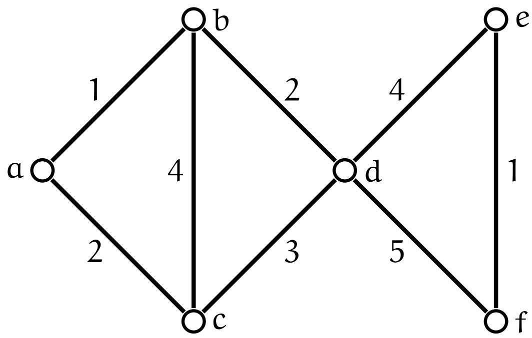
**Problem 2** (2 pts)**.** This problem gives an alternative proof for the existence of spanning trees.

Let G be a connected graph. Consider the family $\mathcal{F}$ of all spanning subgraphs of G that contain no cycle. Show that $\mathcal{F}$ is non empty and that an element of $\mathcal{F}$ with **maximum** number of edges is a spanning tree of G.

**Problem 3** (2 pts)**.** Let G be a connected graph and let S be any subset of edges of G. Show that G has a connected, spanning subgraph H such that $S \subseteq E(H)$ and if C is a cycle in H, then $E(C) \subseteq S$.

**Problem 4** (2 pts)**.** Let G be a connected graph and let $w\colon E(G) \to \mathbb{R}^{>0}$ be a weight function. Show that if all weights are distinct (that is $w(e) \neq w(s)$ for all distinct $e, s \in E(G)$), then G has a *unique* minimum spanning tree.

**Problem 5** (1+ 1 pts)**.** Consider the following weighted graph




(a) Apply Kruskal's algorithm to find a minimum spanning tree of the graph.

(b) Apply the Jarník-Prim algorithm to find minimum spanning tree of the graph.

For each case, include a series of pictures explaining how the minimum spanning tree is created.