Problem 1. Let X be a set. Assume $\mathcal{B}$ and $\mathcal{B}'$ are bases for respective topologies on X. Prove that $\mathcal{T}(\mathcal{B}')$ is finer than $\mathcal{T}(\mathcal{B})$ if and only if for each $B \in \mathcal{B}$ and each $x \in B$, there exists some $B' \in \mathcal{B}'$ such that $x \in B' \subseteq B$.

Problem 2. Consider on $\mathbb{R}$ the standard topology and the lower limit topology. Use the previous problem to decide which of these topologies is finer or coarser than the other.

Problem 3. Let X be a set with more than one element. Prove that $\mathcal{S} = \{X \backslash \{x\} \mid x \in X\}$ is a subbasis for the cofinite topology on X.

Problem 4. Let X and Y be topological spaces with bases $\mathcal{B}$ and $\mathcal{C}$, respectively. Show that the collection

$$\mathcal{D} = \{U \times V \subseteq X \times Y \mid U \in \mathcal{B}, V \in \mathcal{C}\}$$

is a basis for the product topology on $X \times Y$.
Hint: Lemma 13.2 from Munkres.

Problem 5. Endow $\{0, 1\}$ with the discrete topology. Let Λ be a nonempty set. Prove that $\prod_{i \in \Lambda} \{0, 1\}$ (endowed with the product topology) is discrete if and only if Λ is finite.

Problem 6. Give a proof for Theorem 19.3 in Munkres.