Unless explicitly requested by a problem, do not include sketches as part of your proof. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1 (2 pts). (a) For which n is the complete graph K_n Eulerian? When does K_n contain an Eulerian trail?

(b) For which s, t is the complete bipartite graph $K_{s,t}$ Eulerian? When does $K_{s,t}$ contain an Eulerian trail?

Problem 2 (2 pts). Let G be a connected regular graph that is not Eulerian. Show that, if $\overline{G}$ is connected, then $\overline{G}$ is Eulerian. Deduce that the complement of the Petersen graph is Eulerian.

Problem 3 (2pts). Use Eulerian circuits to show that if G is a connected even-regular graph (i.e. all vertices have even degree), then G has no bridges.

Problem 4 (2 + 2 pts). Let G and H be any graphs. Recall that the *Cartesian product* of G and H is the graph $G \times H$ which has vertex set $V(G) \times V(H)$ and $\{(u_1, v_1), (u_2, v_2)\} \in E(G \times H)$ if and only if either $u_1 = u_2$ and $v_1 v_2 \in E(H)$ or $u_1 u_2 \in E(G)$ and $v_1 = v_2$.

(a) Prove that $G \times H$ is connected if and only if both G and H are connected.

(b) Prove that $G \times H$ is Eulerian if and only if both G and H are connected and also:

- Both G and H are even-regular, or
- Both G and H are odd-regular.