Problem 1.

1. Prove that any set endowed with the cofinite topology is compact.
2. When is a set endowed with the cocountable topology compact?

Problem 2. Let X and Y be subspaces of $\mathbb{R}^n$. For each of the following situations, determine whether X and Y can be homeomorphic

1. X closed and bounded, Y closed but not bounded.
2. X bounded, Y non-bounded.
3. X closed and bounded, Y bounded but not closed.
4. X closed, Y not closed.

If you think that such X and Y exist, give an example (and justify). Otherwise prove why no such pairs exist.

Problem 3. Using connectedness and compactness, prove that the following spaces are nor homeomorphic

$$[0,1], \quad [0,1), \quad \mathbb{R}, \quad \mathbb{R}^2, \quad S^1.$$