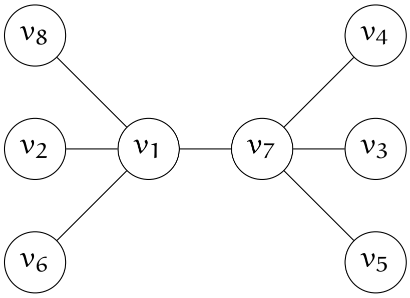
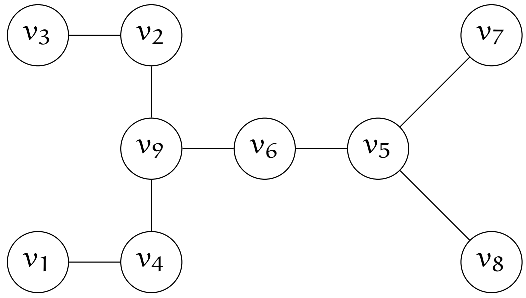
Unless explicitly requested by a problem, do not include sketches as part of your proof. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1 (0.5 + 0.5 pts)**.** Determine the Prüfer code of the following trees, using the ordering $v_1 \leqslant v_2 \leqslant \dots$ for the vertices.

(a)



(b)



Problem 2 (0.5 + 0.5 pts)**.** For the following sequences, draw a picture of the (labeled) tree which has that sequence as its Prüfer code (using the standard ordering of the integers). Your tree should have vertex-set $\{v_1, \dots, v_n\}$ for some integer n.

(a) $(v_5, v_7, v_5, v_1, v_3, v_5, v_5)$

(b) $(v_4, v_4, v_1, v_2, v_1, v_2, v_3)$

Problem 3 (1 pts)**.** Determine (with proof) all trees T (up to isomorphism) on $n \geqslant 2$ vertices whose Prüfer code uses each element of $V(T)$ at most once (under any arbitrary ordering of $V(T)$).

Problem 4 (2 pts)**.** Fix an integer $n \geqslant 2$ and let $d_1, \dots, d_n$ be a sequence of positive integers with $\sum_{i=1}^{n} d_i = 2n - 2$. Use Prüfer codes to show that there is a tree with degree sequence $d_1, \dots, d_n$.

Problem 5 (2 pts)**.** For a graph G, define the relation R on $V(G)$ by $u\,R\,v$ if and only if $u = v$ or there is a cycle in G containing both u and v. Find a graph G wherein R is *not* an equivalence relation on $V(G)$. (You are welcome to define G via a picture, though, of course, you must still demonstrate that R is not an equivalence relation on this G)

Problem 6 (3 pts)**.** For a graph G, define the relation R on $E(G)$ by $e\,R\,s$ if and only if $e = s$ or there is a cycle in G containing both e and s. Prove that R is an equivalence relation on $E(G)$.