Unless explicitly requested by a problem, do not include sketches as part of your proof. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1 (1 pt)**.** Show that if G is a connected graph that is not regular, then G contains **adjacent** vertices u and v such that $\deg u \neq \deg v$.

Hint: there is a reason why this problem is worth 1 pt.

Problem 2 (2 pts)**.** Let G be a graph on n vertices. Prove that if $\Delta(G) + \delta(G) \geqslant n - 1$ then G is connected.

Problem 3 (2 pts)**.** Let G be a graph with the property that $\deg u + \deg v \equiv 1 \pmod 2$ for every $uv \in E(G)$. Prove that $|E(G)|$ is even.

Problem 4 (2 pts)**.** Let G be a connected graph and suppose that $f\colon V(G) \to \mathbb{Z}$ is a function with the property that $f(u) + f(v) \equiv 0 \pmod 3$ for every $uv \in E(G)$. Prove that if G is *not* bipartite, then $f(v) \equiv 0 \pmod 3$ for every $v \in V(G)$.

Problem 5 (2 pts)**.** Let G be a graph on n vertices. Prove the following:

1. $\delta(G) + \delta(\overline{G}) \leqslant n - 1$,
2. $\delta(G) + \delta(\overline{G}) = n - 1$ if and only if G is regular.

Problem 6 (0.5+0.5 pts)**.** Determine whether or not the following sequences are graphical. If the sequence is graphical, draw a picture of a graph with that degree sequences. If the sequence is not graphical, prove this.

1. $5, 3, 3, 3, 3, 2, 2, 2, 1$
2. $6, 5, 5, 4, 3, 2, 1$