Unless explicitly requested by a problem, do not include sketches as part of your proof. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1 (2pts). Prove that $R(3, n) \leqslant n^2$ for every positive integer n.

Problem 2 (2pts). The 3-color Ramsey number $R(m, n, p)$ is the smallest integer N such that every 3-coloring of $E(K_N)$ (say with colors red, blue and green) contains either a red copy of K_m, a blue copy of K_n or a green copy of K_p.

Prove that $R(3, 3, 3) \leqslant 17$.

Problem 3 (2pts). Suppose that G is a graph that has no induced $K_{1,4}$ and such that $\omega(G) = 4$. Show that $\Delta(G) \leqslant 17$.

Hint: $17 + 1 = R(4, 4)$.

Problem 4 (2pts). Show that $N = 7$ is the smallest integer N such that every red,blue-coloring of $E(K_N)$ contains either a red $K_{1,3}$ or a blue K_3.

Problem 5 (2 pts). Let n be a positive integer and set $N = 3n - 1$. Construct a red,blue-coloring of $E(K_{N-1})$ which *does not* contain a monochromatic matching with n edges.

Hint: Break $V(K_{N-1})$ into two pieces, one of size $2n - 1$ and the other of size $n - 1$, and color the edges based on how they intersect these two pieces.