Problem 1. Let X be a topological space and A a subset of X. Define the *interior* of A (denoted A°) and the *closure* of A (denoted $\overline{A}$) by

$$\mathring{A} = \{x \in X \mid A \text{ is a neighborhood of } x\}$$
$$\overline{A} = \{x \in X \mid X \backslash A \text{ is not a neighborhood of } x\}$$

Prove that $\mathring{A} \subseteq A \subseteq \overline{A}$.

Problem 2. Let X be a topological space and A a subset of X. Prove that

$$\mathring{A} = \bigcup_{U \text{ open},\, U \subseteq A} U, \qquad \overline{A} = \bigcap_{C \text{ closed},\, C \supseteq A} C.$$

Problem 3 (10 points)**.** Let X be a topological space and A a subset of X.

1. Prove that $\mathring{A}$ is the largest open set contained in A and that $\overline{A}$ is the smallest closed set containing A.

2. Prove that A is open if and only if $A = \mathring{A}$ if and only if A is a neighborhood of each of its points.

3. State and prove the closed version of the first biconditional (if and only if) of Part 2.

4. For the standard topology on $\mathbb{R}$, compute the interior and the closure of the interval $[0, 2)$.

Problem 4. Let X be any set. Prove that the family of subsets

$$\mathcal{T}_{cf} = \{U \subseteq X \mid X \backslash U \text{ is finite } \} \cup \{\varnothing\}$$

defines a topology on X. This is called the *cofinite* topology.

Problem 5. Let X be a topological space and let A be a subset of X. Prove that the family of subsets

$$\mathcal{T}_A = \{V \subseteq A \mid \text{there exists an open set } U \subseteq X \text{ such that } V = U \cap A\}$$

defines a topology on A. This is called the *induced* topology.

Problem 6. Munkres 13.3

Problem 7. Munkres 13.4

Problem 8. Munkres 13.6

Problem 9. Munkres 13.8